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Make $n=0$, and $A=\frac{1}{2}+\frac{1}{6}\frac{r}{2}-\frac{1}{30}\frac{r^3}{4}+\ldots$

Hence the value of n years annuity at simple interest, r per £1, is

$$\frac{1}{1+r}+\ldots+\frac{1}{1+nr}=\frac{1}{r}\log(1+nr)-\tfrac{1}{2}\left(1-\frac{1}{1+nr}\right)+\tfrac{1}{6}\frac{r}{2}\left(1-\frac{1}{(1+nr)^2}\right)$$
$$-\tfrac{1}{30}\frac{r^3}{4}\left(1-\frac{1}{(1+nr)^4}\right)+\tfrac{1}{42}\frac{r^5}{6}\left(1-\frac{1}{(1+nr)^6}\right)-\tfrac{1}{30}\frac{r^7}{8}\left(1-\frac{1}{(1+nr)^8}\right)$$
$$+\tfrac{5}{66}\frac{r^9}{10}\left(1-\frac{1}{(1+nr)^{10}}\right).$$

Multiply by r, for r write n^{-1}, and we have

$$\frac{1}{n+1}+\ldots+\frac{1}{2n}=\log 2-\frac{1}{4n}+\tfrac{1}{6}\frac{1}{2n^2}\left(1-\frac{1}{2^2}\right)-\tfrac{1}{30}\frac{1}{4n^4}\left(1-\frac{1}{2^4}\right)+\ldots$$

Add this to the formula already found for $1+\ldots+\frac{1}{n}$, and we see, as it ought to be, that the result is that formula with n changed into $2n$.

On a Table for the Formation of Logarithms and Anti-Logarithms to Twelve Places. By PETER GRAY, *F.R.A.S., Honorary Member of the Institute of Actuaries.*

PART IV.—*History of the Method.*

SO far as I am aware, the earliest method for the formation of logarithms, in which it was proposed to resolve the number whose logarithm is required into factors *by a continuous process,* is one which was published by Mr. Manning in the *Philosophical Transactions* for 1806. My acquaintance with Mr. Manning's method is derived, not from the original paper, which I have not seen, but from a reprint of it, "nearly as it stands," in Young's *Elementary Essay on the Computation of Logarithms.** Mr. Manning applies his method to the formation of Naperian logarithms only; but it is equally applicable to the formation of common logarithms. I repeat, in accordance with this method, an example I have already more than once given.

Required the logarithm of π.

The following is the process:—

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* Second edition, London, 1835, pp. 67–79.

π=3·141592653590	÷2
1·57079632679~~5~~	
157079632680	
1·41371669411~~5~~	
141371669412	
1·27234502470~~3~~	
127234502470	
1·14511052223~~3~~	
114511052223	
1·0305994700~~1~~0	4
10305994700	
1·0202934753~~1~~0	
10202934753	
1·010090540~~5~~57	2
1010090541	
1·009080450~~0~~16	
1009080450	
1·008071369~~5~~66	
1008071370	
1·007063298~~1~~96	
1007063298	
1·006056234~~8~~98	
1006056235	
1·005050178~~6~~63	
1005050179	
1·004045128~~4~~84	
1004045128	
1·003041083~~3~~56	
1003041083	
1·002038042173	

1·002038042~~2~~73	
1002038042	
1·001036004~~2~~31	
1001036004	
1·0000349~~0~~8227	10
10000350	
1·0000249~~0~~7877	
10000250	
1·0000149~~0~~7627	
10000150	
1·000004~~9~~67477	3
1000005	
1·000003~~9~~67472	
1000004	
1·000002~~9~~67468	
1000003	
1·000001~~9~~67465	
1000002	
1·00000~~0~~967463	4

301029995664	log 2		
183029962243	4	Col.	1
8729610805	2	,,	2
4345117740	10	,,	3
13028900	3	,,	5
1737179	4	,,	6
390865	9	,,	7
26058	6	,,	8
3040	7	,,	9
174	4	,,	10
26	6	,,	11
1	3	,,	12
0·497149872695	=log π.		

The resolving process here, after preparation of the number as in the method already exemplified, consists obviously of series of subtractions, in each of which the subtrahend bears a definite and easily recognisable ratio to the minuend. In the first series, consisting of *four* subtractions (as indicated in the margin), this ratio is 1 : 10; in the second (*two* subtractions), the ratio is 1 : 100; in the third (*ten** subtractions), 1 : 1,000; in the fourth (*three* subtractions), 1 : 100,000; and in the fifth (*four* subtractions), 1 : 1,000,000. The gradation amongst the ratios will be observed. The ratio 1 : 10,000 is missing; and the omission will be found to correspond to the passing over of a column in the subsequent part of the operation.

The object aimed at in the successive subtractions is the exhaustion of the decimal portion of the minuend, and the consequent reduction of the last remainder to unity. A subtrahend, therefore, greater than this decimal portion, is inadmissible; and it is when the next subtrahend would be in this condition that the necessity for a change in the ratio arises. And the point at which the change is requisite is indicated also usually but not universally (see in the example the change from 1 : 100 to 1 : 1,000) by the exhaustion, by the immediately preceding subtraction, of the figure occupying the highest place in the number to be exhausted.

When the first half of the decimal portion has been exhausted, that is, when the minuend has been reduced to unity followed by six ciphers, it is unnecessary to carry the process further, as the results we should obtain are already visible. The ratios of the several subtrahends to their minuends are those that follow by natural sequence the ratios already noted, and the number of subtractions in the several sums are indicated respectively by the figures composing the unexhausted portion of the minuend.

The second part of the operation shows the formation of the logarithm of π. The marginal numbers and their continuation in the last remainder form the arguments for the several columns; and the corresponding results, added to the logarithm of the preparing number, give the logarithm required.

The rationale of the foregoing process is as follows:—To subtract from a number one-tenth of itself is equivalent to multiplication of it by $\frac{9}{10}$. In like manner, subtraction of $\frac{1}{100}$ is equivalent to multiplication by $\frac{99}{100}$, and so on. But such multiplications are also equivalent to divisions by the reciprocals of the several multipliers respectively. It hence appears that the operations performed on π, in the example before us, are in effect one division by 2, four by $\frac{10}{9}$, two by $\frac{100}{99}$, ten by $\frac{1000}{999}$, and so on, the last quotient being unity, for the remainders are the successive quotients, as the process is now looked at. Hence we have the given number resolved into factors as follows:—

---

* Mr. Manning (or Mr. Young) says, that the number of subtractions in any series can "never exceed nine." Here, in an example casually taken, is evidence to the contrary.

$$\pi=2\times\left(\frac{10}{9}\right)^4\times\left(\frac{100}{99}\right)^2\times\left(\frac{1000}{999}\right)^{10}\;.\;.\;.\;.\;.$$

$$\therefore \log\pi=\log 2+4\log\frac{10}{9}+2\log\frac{100}{99}+10\log\frac{1000}{999}+\;.\;.\;.\;.$$

Hence a table* containing the first nine (or ten) multiples of $\log\frac{10}{9}$, $\log\frac{100}{99}$, &c., will furnish the logarithms requisite for the application of this method.

The preceding is the manner in which his method was presented by Mr. Manning. It admits of being presented in a different manner, as an application of a general principle, which includes also other methods, to be immediately described. The principle in question may be enunciated as follows:—

If, disregarding the decimal point, any given number be continuously multiplied by a series of factors till a result is obtained consisting of unity followed by n ciphers, the product of those factors will be the reciprocal of the given number, true to $n+1$ places.

Hence the sum of the logarithms of the factors will be the arithmetical complement of the logarithm of the given number; or, the sum of the logarithms of the reciprocals of the factors will be the logarithm of the given number.

The foregoing principle is too obvious to require a word to be said in explanation of it. In applying it, the point to be principally attended to is to arrange for the employment of factors which shall be easy to use, and whose logarithms can be readily formed and tabulated. In the foregoing example of Mr. Manning's process, the series of factors by which π is multiplied is

$$\tfrac{1}{2},\ \frac{9}{10}(4\text{ times}),\ \frac{99}{100}(\text{twice}),\ \frac{999}{1000}(10\text{ times}),\ \&\text{c.};$$

and the result is unity followed by twelve ciphers. Hence the sum of the cologarithms of these factors is the logarithm of π.

Mr. Manning's factors may be written as follows:—

---

* I have in my example *supposed* the data arranged in columns. They were not so arranged by Mr. Manning. And I may mention, that although I have here, as in previous examples, indicated a number of columns equal to that of the *possible* tabular entries, and shall follow the same course in subsequent examples, yet, in all cases with which we have had or shall have to do, the second half of the entries are obtained from a single column. (See note, p. 212.)

$$\frac{9}{10}=\frac{10-1}{10}=1-\cdot1,\quad \frac{99}{100}=\frac{100-1}{100}=1-\cdot01,$$

$$\frac{999}{1000}=\frac{1000-1}{1000}=1-\cdot001,$$

and so on, the general form being $1-\cdot1^m$, or $1-\cdot1^m n$, where n is always unit. It does not appear whether Mr. Manning recognised the general principle above stated. Very probably he did not; but I think it is unquestionable that he did not put his factors into the form here given, since, had he done so, it is inconceivable that he should have failed to see that there is no necessity for the restriction of n to the value unity. To do him justice, however, he seems to have presented his method less as a *practical* method for the formation of logarithms generally, than as one for the formation of the fundamental logarithms preparatory to the construction of a complete table.

Mr. Manning's process, as we have seen, although beautifully simple, is also very laborious, and, for this reason, probably, it does not seem for many years to have attracted much attention, or to have undergone any improvement. At length, in Professor Young's reprint of Mr. Manning's paper, it came under the notice of Mr. Weddle, then of Newcastle, and subsequently of the Royal Military College, Sandhurst, who, in *The Mathematician* for Nov., 1845, pp. 17–25, reproduced it, with the improvement in the resolving process above hinted at, and also an extension to the converse problem of finding the number corresponding to a given logarithm. I repeat the example just given, worked according to Mr. Weddle's process.

$\pi=3\cdot141592653590$	$\div 2$
$1\cdot57079632679\cancel{5}$	3
471238898039	
$1\cdot0995574287\vert\cancel{5}6$	9
98960168588	
$1\cdot00059726\cancel{0}168$	05
500298630	
$1\cdot0000969\cancel{0}1538$	9
90008726	
$1\cdot000006\cancel{9}52812$	6
6000042	
$1\cdot00000\cancel{0}952770$	

301029995664	log 2		
154901959986	3	Col.	1
40958607679	9	„	2
217201546	5	„	4
39088262	9	„	5
2605775	6	„	6
390865	9	„	7
21715	5	„	8
869	2	„	9
304	7	„	10
30	7	„	11
0	0	„	12
$0\cdot497149872695$	log π		

The abbreviation here is manifest, the number of subtractions being reduced from twenty-three to five; and it will be found that it has not been purchased at an undue cost. The principle employed in the process is, as in Mr. Manning's process, that above enunciated, the given number being continuously multiplied till a product is attained consisting of unity followed by twelve ciphers.* The factors (after the first) are of the form $1 - \cdot 1^m n$, in which m takes the values 1, 2, 3, &c., successively, and n takes the values which in each case, when multiplied into the remainder, will give the greatest product not exceeding the decimal portion of that remainder. The operation consequently becomes assimilated to that of division, the dividend at any one point being the decimal portion of the remainder, the divisor the entire remainder, and the quotient a value of n.

π is here shown to be the reciprocal of the following product:—

$$\tfrac{1}{2} \times (1 - \cdot 3)$$
$$\times (1 - \cdot 09)$$
$$\times (1 - \cdot 0005),\ \text{and so on.}$$

And hence the sum of the *cologarithms* of the component factors is the logarithm of π.

It is the cologarithms of these factors that Mr. Weddle tabulates,† and his anti-logarithmic method takes its form in accommodation to that of his tabulated values. I here present an example of it.

log π =	0·497149872694				·000000376405	
ar. co. =	502850127306	log. 2			999999$\cancel{6}$23595	7
	301029995664				6999997	
	201820131642	3	Col.	1	9999926$\cancel{7}$3598	6
	154901959986				59999557	
	46918171656	9	,,	2	99993262$\cancel{4}$041	6
	40958607679				599959574	
	5959563977	1	,,	,,	999332664$\cancel{4}$67	3
	4364805402				2997997993	
	1594758575	3	,,	3	996334666474	1

---

* As in Mr. Manning's method, so also here, it is unnecessary formally to carry the resolving process beyond the exhaustion of the first six decimal places; since at that point what would be the result of its formal completion is visible in the remainder.

† He arranges them, not in columns, as the indications I have attached to the preceding example would seem to imply, but in an unbroken series. There are other breaches of continuity in the method as delivered by Mr. Weddle, which also I have removed.

1594758575 1304841688	3	Col. 3
289916887 260654893	6	„ 4
29261994 26058451	6	„ 5
3203543 3040072	7	„ 6
163471 130288	3	„ 7
33183 30401	7	„ 8
2782 2606	6	„ 9
176 174	4	„ 10
2 2	5	„ 12
–		

996334666474 9963346665	1
986371319809 88773418782	9
897597901027 269279370308	3
628318530719	÷2
3·141592653595 = π	

On the left is shown the decomposition of the arithmetical complement of the given logarithm into tabular logarithms, and ar-co. log π, or log $(1 \div \pi)$, is shown to be equal to

$$\log 2 + \log \frac{1}{1-{\cdot}3} + \log \frac{1}{1-{\cdot}09} + \log \frac{1}{1-{\cdot}01} + \ldots ;$$

whence, taking the complements,

$$\log \pi = \log \tfrac{1}{2} + \log(1-{\cdot}3) + \log(1-{\cdot}09) \ldots$$

and therefore

$$\pi = \tfrac{1}{2} \times (1-{\cdot}3) \times (1-{\cdot}09) \times \ldots$$

The operation on the right shows the formation of π by the multiplication of the factors determined as above. It needs a little elucidation. The last six factors (Col. 11 is wanting) are 1 − ·0000003, 1 − ·00000007, &c., and their product, in the same mixed form, is 1 − ·000000376405, the negative portion of which therefore can be written down at once, by inspection.* Subtracting this from unity, we get for the same product, in a positive form, ·999999623595; and this being multiplied by the other factors in succession, the result is the value of π. The factors last referred to are used here in reverse order. They may, however, if we please, be used in direct order; in which case the compounding process would be in entire analogy—in fact identical—with the resolving process.

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* See page 219.

Mr. Weddle's paper came under my notice in the early part of 1846, and I was immediately attracted by the method as there developed. I soon perceived that it admitted of several modifications, by which its efficiency and the facility of its application would be increased; and, in particular, I observed that by the formation of a new table, adapted to two-figure values of *n*, the number of tabular entries requisite in any particular case would be reduced one-half. I therefore constructed such a table, to twelve places; and I communicated an account of the method, with the new table and my other improvements, to the *Mechanics' Magazine.** My communication appeared in the numbers for October and November, 1846.

I repeat the last example, worked by my two-figure table:—

$\pi=$	3·1415 9265 3590	÷2
	1·5707 9632 679$\cancel{5}$	36
	4712 3889 8039	
	995 5742 8756	
	942 4777 9608	
	1·0053 0964 9$\cancel{1}$48	52
	50 2654 8246	
	2 8310 0902	
	2 0106 1930	
	1·0000 820$\cancel{3}$ 8972	82
	8000 6563	
	203 2409	
	200 0164	
	03 2245	

3010 2999 5664	log 2	
1938 2002 6016	36	Col. 1
22 6422 3402	52	„ 2
3561 3608	82	„ 3
1 3029	03	„ 4
955	22	„ 5
20	45	„ 6
0·4971 4987 2694	=log π	

The above needs little explanation. The quotients being extended to two figures, π is resolved into the continued product following:—

---

* I am glad to have an opportunity of placing it upon record that Mr. Weddle, with whom I was not previously acquainted, called upon me and thanked me very cordially for the improvements I had introduced in his method, and for my endeavours to bring it more prominently before the public.

$$\begin{aligned}2&\times(1-\cdot 36)^{-1}\\ &\times(1-\cdot 00{,}52)^{-1}\\ &\times(1-\cdot 00{,}00{,}82)^{-1}\\ &\times(1-\cdot 00{,}00{,}00{,}03)^{-1}\\ &\times(1-\cdot 00{,}00{,}00{,}00{,}22)^{-1}\\ &\times(1-\cdot 00{,}00{,}00{,}00{,}00{,}45)^{-1};\end{aligned}$$

and the logarithms of the factors composing it being taken from the tables, their sum is the logarithm required. The abbreviation arising from the use of the extended table is chiefly apparent in the second part of the operation, as comparison with the corresponding operation on p. 256 will show.

I now exhibit the anti-logarithmic process:—

log π =	4971 4987 2694		0000 0017 8739	
ar. co. =	5028 5012 7306	log 2	9999 9982 126$\cancel{1}$	37
	3010 2999 5664		2999 9994 6378	
	2018 2013 1642	37 Col. 1	6999 9987 4883	
	2006 5945 0546		699 9998 7488	
	11 6068 1096	26 „ 2	6299 9988 7$\cancel{3}$95	26
	11 3063 6118		12 5999 9775	
	3004 4978	69 „ 3	87 3988 7620	
	2996 7353		3 7799 9932	
	7 7625	17 „ 4	6283 618$\cancel{8}$ 7688	69
	7 3830		3770 1713	
	3795	87 „ 5	2418 5975	
	3778		565 5257	
	17	39 „ 6	6283 1853 0718	÷2
	17			
	—		3·1415 9265 3590	$=\pi$

On the left is the decomposition of colog π into tabular values, which being the co-logarithms of $\frac{1}{2}$, $(1-\cdot 37)$, $(1-\cdot 00{,}26)$, $(1-\cdot 00{,}00{,}69)$, &c., multiplication of these factors, as on the right, gives π. It will be perceived that the product of the last three factors is formed as in the last example, and the result multiplied by the remaining factors in *direct* order.

The operation now is a very compact one, and it might be rendered a little more so by the formation of a three-figure table adapted to it. It has its weak points, however. These are, first, that the multipliers occasionally *overlap* each other, rendering two entries in the same column necessary, as in the example on pp. 257, 258;

and secondly, that the anti-logarithmic operation partakes somewhat of an indirect character. But these are only slight blemishes; and had not another method which is free from them been found, I think there is no doubt that, had time been allowed, Mr. Weddle's would have been received as the best practical method that had as yet been devised for the formation of logarithms and anti-logarithms when more places are wanted than are afforded by the common tables. I proceed to give an account of the origination of the method now referred to, which is that which I have sought to develop, and to which the accompanying tables are adapted.

In the *Mathematician* for March, 1847, appeared a paper by Mr. Hearn, of the Royal Military College, Sandhurst, intituled, "Practical method of forming logarithms and anti-logarithms, independently of extensive tables." Mr. Hearn's logarithmic method, as described in his paper, was identical with Mr. Weddle's.* It, therefore, needs no further notice here. His anti-logarithmic method, however, is altogether different. It is the one that I have appropriated; and the example on pp. 132, 133, may be accepted as an illustration of it, although Mr. Hearn's table extended to only ten places, and the operation as here given is considerably systematized.

In the *Mechanics' Magazine* for February 12th, 1848, I gave an account and exemplification of Mr. Hearn's anti-logarithmic method, and also a table, to twelve places, adapting it to the two-figure process. The advantage gained in point of abbreviation and facility by this extension of the data, will be appreciated on comparison of the example on pp. 132, 133, already referred to, with that on p. 135, which latter is worked by the table which accompanied my paper. My paper, I perceive, is dated December 21st, 1847, very soon after which date I discovered the applicability of the table to the direct operation—that of forming the logarithms of numbers, and showed it to my friend, the late Mr. Woollgar, and others. On the 7th February following, in a private note, I informed Mr. Robertson, the Editor of the *Mechanics' Magazine*, of the discovery, and asked for space to explain it in his pages. In his reply, dated on the 14th, he stated that he had received a paper in which the adaptation of the table to which I referred was shown, and offered to publish along with that paper whatever I might have to communicate on the subject. I immediately, on the 15th, wrote to Mr. Robertson, communicating an example of my method, and both letters appeared in the Number of the Magazine for February 26th. Mine bore my own name, and the other had attached to it the initials W. O. The following are the examples of the two methods as they appeared in the papers to which reference has just been made. The first is mine, and the second is W. O.'s :—

---

* It was singular, to say the least, to find given as original, and without remark, a method which had appeared in the pages of the same periodical sixteen months before. I had some correspondence with Mr. Hearn on the subject of his paper, in the course of which he informed me of the circumstances connected with the preparation and publication of it. The account he gave me was confirmed by a friend of his own, a gentleman of unquestionable veracity; and also, subsequently, so far as they were cognisant of the circumstances, by the Editors of the *Mathematician*. The circumstances were as follows:—The paper was prepared not later than September, 1845, at least two months before the appearance of Mr. Weddle's paper. In November of the same year it was handed to one of the Editors of the *Mathematician;* and the identity of the logarithmic methods developed in the two papers having been recognised, Mr. Hearn accompanied it with the requisite explanation. For some reason unexplained the paper did not appear till March, 1847, and the explanation was not given. It was given subsequently, as already hinted; but the omission of it at the proper time exposed Mr. Hearn to a charge of plagiarism, which is now seen to have been altogether undeserved.

It is right to add that Mr. Weddle's paper is dated August 18, 1845; and that he states in it that his methods "were discovered nearly seven years ago."

	3·14,159,2,65359,0	3 × 104
	3 12	
3 1 2)	2 159	69
1 8 7 2	1 872	
	287 2	
2 8 0 8	280 8	
3 1 4 1 5 2 8)	6 4 65359	20
6 3	6 2 83056	
3,1,4,1,5,9,1)	1 82303 0	58
	1 57079 6	
	25223 4	
	25132 7	
	90 7	02
	62 8	
	27 9	89
	25 1	
	2 8	
	2 8	

477121254720	log 3
17033339299	04
2986340857	69
8685803	20
251891	58
87	02
39	89
·497149872696	=log. required.

3141,5926,5359,0	×3		
9424,7779,6077,0	06		
565,4866,7764,6			
		Log 3	=4771,2125,4720
9990,2646,3841,6	09	Col. 1 06=	253,0586,5265
8,9912,3816,5		Col. 2 09=	3,9068,9250
		Col. 3 74=	3213,6603
9999,2558,7658,1		Col. 4 41=	17,8061
6999,4791,1	74	Col. 5 78=	3388
399,9702,3		Col. 6 48=	21
9999,9958,2151,5			5028,5012,7308
41,7848,5	ar. com.	ar. com.	=4971,4987,2692=required log.

My process differs but little in form from that which I now use, as will be apparent on comparison of it with the two-figure examples on p. 128, with the first of which, indeed, it has much in common. It therefore needs no further explanation.

The second process—that of W. O.—depends, like those of Mr. Manning and Mr. Weddle, on the principle laid down on p. 255, the distinction being that the factors employed by W. O. are of the form $1+n$, in consequence of which additions take the place of subtractions. The proper multiplier in each case is the greatest number that can be employed, so that the sum of the multiplicand and the new product shall not exceed unity; it is, therefore, usually, after the first step, the complement to nine, of the first figure in the multiplicand that differs from nine. As in Mr. Manning's and Mr. Weddle's processes, the formal operation here ceases when the first half of the quotient figures (as I shall call them) have been determined, and the remaining half are the complementary figures of the latter portion of what would be the next multiplicand. The foregoing process being proposed for use with my two-figure table, which contains the logarithms of the factors employed, the sum of the tabular results corresponding to the several multipliers is the cologarithm of the given number, from which the logarithm can, of course, be immediately deduced.

In illustrating, as I have done, and presenting for adoption my own method, I have sufficiently indicated my preference of it to that of W. O. It is natural I should be partial to a method originating with myself; but it will be expected that I should be able to justify my preference on other grounds. And I believe I can do so. While in regard to efficiency and the number of figures employed the two methods may be admitted to be on an equality, I claim for my own superiority in point of simplicity and facility. In it the resolving process differs but little from the already familiar process of ordinary division, becoming more closely assimilated to that process as the table is extended for the purpose of reducing the number of tabular entries. The corresponding process in W. O.'s method, on the other hand, is of a mixed character, requiring a considerable amount of care and attention for the avoidance of error; and it is in no degree improved by the extension of the table. All this will be sufficiently apparent if an example be worked, using the three-figure table. W. O.'s method also is somewhat indirect: it is the cologarithm of the given number that is first found, from which the logarithm has to be deduced.*

In an operation such as the present, the necessity for which arises only now and then, revision of the rules for its performance may occasionally be necessary when the operation presents itself. And for this reason, irrespective of the more obvious one, the greatest attainable simplicity in both the operation itself and the description of it is desirable.

The author of the method brought, as above, into comparison with mine—the owner of the initials W. O.—was the late William Orchard, well known in actuarial circles as the author of a useful and meritorious work on Assurance Premiums. He introduced himself to me just after the simultaneous publication of our methods in the *Mechanics' Magazine;* and we found when we met, that, besides the subject that had brought us together, there were a number of others in which we felt a common interest. Of those one to which Mr. Orchard had devoted much attention was the subject of life contingencies. He published his work above referred to in 1850, and in 1851 he became one of my coadjutors in the preparation of a work consisting of results deduced from the Carlisle table of mortality.†

Mr. Orchard died in 1854, at the age of thirty; and Mr. Weddle and Mr. Hearn having also died in their early prime, has fallen to me, in the foregoing historical sketch, to endeavour to apportion to each his share in the origination and development of the methods which have culminated, as I conceive, in that which is presented for adoption in the preceding pages. I have sought to do so impartially, and to the best of my information and ability; and I hope it will not be found that in what I have written I have afforded cause for complaint to the friends of any of the gentlemen whose claims I have had to discuss.

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* This indirectness would be got rid of if the cologarithms instead of the logarithms of the factors were tabulated. But it would then appear in the converse process, as does in Mr. Weddle's method. There is no avoiding it, in the one operation or the other, in any method making use of the principle laid down on p. 255.

† Further particulars regarding Mr. Orchard are contained in an Introduction, prepared by me, to a re-issue of his work in 1856.

It only remains now that I should mention, that in Captain (now Major-General) Shortrede's *Logarithmic Tables** (1849), all the methods I have exemplified (with the exception of Mr. Manning's) are illustrated; and one and two-figure tables, of both Weddle's and Hearn's forms, are given for their application—the two-figure tables extending to 16 places and the one-figure to 25.

On Government Annuity and Assurance Rates and Regulations. By MARCUS N. ADLER, M.A., *of the Alliance Assurance Office, Fellow of the Institute of Actuaries and of the Statistical Society.*

[Read before the Institute, the 24th April, 1865.]

IN briefly commenting upon the papers that have been presented to Parliament on the subject of life assurances and annuities now granted by Government, it is less my object to criticize the measures that have been adopted, than to explain the mode of construction of the several tables of rates that are being acted upon. In consequence of several assumptions and suppositions, these have not been computed in the mode usually adopted by actuaries.

I may remind the members, that last year, when the Bill which has now passed into law was before Parliament, I had the honour of reading a paper before the Institute on this subject, and it is a source of satisfaction to me, that most of the suggestions contained in that paper appear to have been adopted. I feel particularly gratified that an Act was passed towards the end of the last Session, the 27 & 28 Vict., cap. 56, by which Probates or Letters of Administration for amounts not exceeding one hundred pounds, are

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* Will it be considered out of place if I here call the attention of computers to this very useful work? In a handy volume it contains, besides sundry smaller tables, the ordinary seven-figure logarithmic table, and also an anti-logarithmic table of like extent. The convenience of the latter table, when many numbers to logarithms of seven places have to be taken out, is hardly conceivable by those who have not made trial of it. It is a great advantage also to have both tables in a single volume. The price of the book is, I believe, only twelve shillings.